ON THE STRONG CONSISTENCY OF APPROXIMATE MAXIMUM LIKELIHOOD ESTIMATORS

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1. Introduction and statement of problem

Wald's general conditions for strong consistency of Approximate Maximum Likelihood Estimators (AMLE) [11] have been extended by several authors, notably LeCam [9], Kiefer and Wolfowitz [8], Huber [7], Bahadur [1], and Crawford [4]. Except for mild identifiability and local regularity conditions these papers (except [9]) share two critical global assumptions, global in the sense that they concern the behavior of the Log Likelihood Ratio (LLR) over the entire parameter space Θ (which may be infinite dimensional). Crudely stated these are (a) there exists a "suitable compactification" $\bar{\Theta}$ of Θ (see [1], p. 320) to which the LLR may be extended in a continuous manner without altering the value of its supremum, and (b) the supremum of the LLR is integrable (dominance). Condition (b), however, is not satisfied in many common problems, especially multiparametric ones, where AMLE are known to be consistent. Kiefer and Wolfowitz and later Berk [3] suggested a method which seemed to overcome this difficulty in special cases, namely: consider the observations pairwise, or in groups of k. In the more general context of "maximum w" estimation described below, however, this method fails (see Example 2). Noticing this, Huber proposed that the LLR be divided by a function $b(\theta)$ such that this normalized LLR satisfies (a) and (b).

In Section 2 of this paper we show that under an extended global dominance assumption the method of Kiefer, Wolfowitz, and Berk is precisely the correct one. This idea is then extended to include Huber's modification. In Section 5 we show that (generalized versions of) LeCam's conditions are equivalent to those based on dominance.

The methods described above have several drawbacks, however: they require determination of a suitable group size k, normalizing functions $b(\theta)$, and a compactification $\bar{\Theta}$. As demonstrated by several examples below, these are not always naturally occurring quantities and may be difficult to determine. In Section 3 we introduce a new condition for strong consistency, based on a global uniformity assumption rather than dominance, which seems to present a more natural and straightforward method for determining strong consistency.

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