LIMITS OF EXPERIMENTS

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1. Introduction

In a recent paper J. Hájek [4] proved a remarkably simple result on the limiting distributions of estimates of a vector parameter θ. It turns out that this result, as well as many of the usual statements about asymptotic behavior of tests or estimates, can be obtained by a general procedure which consists roughly in passing to the limit first and then arguing the case for the limiting problem. This passage to the limit relies on some general facts which are perhaps not entirely elementary. They depend heavily on the techniques of L. LeCam [8]. However, these general facts are of interest by themselves. If they are taken for granted the basic result of Hájek [4] and many results of A. Wald [13] become available immediately.

The present paper is organized as follows. Section 2 recalls a number of definitions and theorems which are variations on those given by the author in [8]. We have used here again a simplified definition of "experiments" barely different from the one given in [8]. There is no essential difficulty in returning to the more usual description, at least under appropriate restrictions. However the simplified (or "more abstract" as is claimed by some) description avoids measure theoretic technicalities and makes the arguments more transparent.

Section 3 gives further theorems concerning experiments with a fixed set of indices. It uses the metric introduced in [8] to define a weak topology on the space of experiments indexed by a given set Θ. Although the compactness statements proved in this section are not absolutely essential to the remainder of the paper, they do produce a number of simplifications.

The metric of [8] was intended, in part, to insure a certain continuity of risk functions, at least if loss functions stay bounded. The purpose of Section 4 is to show that a similar type of lower semicontinuity still exists for the weak topology of Section 3, even if the loss functions are only bounded from below.

In Section 5 we consider two types of limits: (1) limits of experiments in the weak topology of Section 3, and (2) experiments formed by taking limiting distributions of certain statistics. The main result is that the experiments of second type are always weaker than those of the first type. Statistics systems for which the two coincide are characterized. Another result of Section 5 is the existence of transitions which are convolutions in the case of shift invariant experiments. For this see also E. Torgersen [12] and H. Heyer [6].

Section 6 elaborates a few examples indicating some of the results implied by the previous propositions. It reproduces partially some results of Hájek, [4] and