APPLICATIONS OF CONTIGUITY TO MULTIPARAMETER HYPOTHESES TESTING

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1. Summary and introduction

Consider a Markov process whose probability law depends on a k dimensional ($k \geqq 2$) parameter θ. The parameter space Θ is assumed to be an open subset of R^k. For each positive integer n, we consider the surface E_n defined by $(z - \theta_0)'\Gamma(z - \theta_0) = d_n$ for some sequence $\{d_n\}$ with $0 < d_n = O(n^{-1})$; Γ is a certain positive definite matrix.

For testing the hypothesis $H: \theta = \theta_0$ against the alternative $A: \theta \neq \theta_0$, a sequence of tests is constructed which, asymptotically, possesses the following optimal properties within a certain class of tests. It has best average power over E_n with respect to a certain weight function; it has constant power on E_n and is most powerful within the class of tests whose power is (asymptotically) constant on E_n. Finally, it enjoys the property of being asymptotically most stringent.

In this paper, we are dealing with the problem of testing the hypothesis $H: \theta = \theta_0$ when the underlying process is Markovian. The parameter θ varies over a k dimensional open subset of R^k denoted by Θ. Since the alternatives consist of all $\theta \in \Theta$ which are different from θ_0, one would not possibly expect to construct a test whose power would be "best" for each particular alternative. Therefore interest is centered on tests whose power is optimal over suitably chosen subsets of Θ. The class of subsets of Θ considered here consists of the surfaces of ellipsoids centered at θ_0. The question then arises as to which restricted class of tests one could search and still obtain an optimal test. The discussion detailed in Section 5 produces a class of tests, denoted by $\bar{\mathscr{F}}$, which consists of those tests each of which is the indicator function of the complement of a certain closed, convex set. The precise definition of $\bar{\mathscr{F}}$ is given in (4.4) and the arguments leading to it are due to Birnbaum [1] and Matthes and Truax [14]. The main steps of these arguments are summarized in an appendix for easy reference.

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