SOME ASYMPTOTIC PROPERTIES OF LIKELIHOOD RATIOS ON GENERAL SAMPLE SPACES

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1. Introduction

It is shown in [2] (see also [3], [4], [14]) under certain conditions that in the case of independent and identically distributed observations likelihood ratios are asymptotically optimal test statistics in the sense of exact slopes. The present paper points out that many of the arguments and conclusions of the papers just cited extend to general sampling frameworks, and also develops certain refinements of these conclusions. The present generalizations and refinements seem worthwhile for the following reasons. They enable us to construct asymptotically optimal tests in problems such as testing independence in Markov chains and in exchangeable sequences. Secondly, they provide a useful method of finding the exact slope of a statistic which is equivalent, on some sample space, to the likelihood ratio on that sample space. It suffices in this case to evaluate the limit, in the nonnull case, of the normalized log likelihood ratio; it is not necessary to obtain estimates of the relevant large deviation probabilities in the null case; indeed, the latter estimates are implicit in the initial evaluation. Finally, the present elaborations throw some light on what sort of conditioning is advantageous in making conditional tests. It seems that a conditioning statistic is helpful if it produces an exact conditional null distribution for the contemplated test statistic *and* if in the testing problem on hand the conditioning statistic is useless by itself.

The following Sections 2 to 4 describe the general theory. Some examples illustrative of the theory are given in Sections 5 to 7.