EXAMPLES OF EXPONENTIALLY BOUNDED STOPPING TIME OF INVARIANT SEQUENTIAL PROBABILITY RATIO TESTS WHEN THE MODEL MAY BE FALSE

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1. Summary and introduction

Two examples are presented (one of them the sequential χ^2 test) of parametric models in which the invariant Sequential Probability Ratio Test has exponentially bounded stopping time N, that is, satisfies $P(N > n) < \rho^n$ for some $\rho < 1$, where the true distribution P may be completely arbitrary except for the exclusion of a certain class of degenerate distributions. Another example demonstrates the existence of P under which N is not exponentially bounded, but even for those P we have $P(N < \infty) = 1$. In the last section a proof is given of the representation (2.1), (2.2) of the probability ratio R_n as a ratio of two integrals over the group G if G consists of linear transformations and translations.

Let $Z_1, Z_2, \cdots$ be independent, identically distributed (i.i.d.) random vectors which take their values in d dimensional Euclidean space E^d and possess distribution P. The symbol P will also be used for the probability of an event that depends on all the Z_i. Let Θ be an index set (parameter space) such that for each $\theta \in \Theta$, P_θ is a probability distribution on E^d. We shall say "the model is true" if the true distribution P is one of the P_θ, $\theta \in \Theta$, but it should be kept in mind throughout that we shall also consider the possibility that the model is false, that is, that P is not one of the P_θ. In the latter case we shall also speak of P being outside the model as opposed to P being in the model. Let Θ_1, Θ_2 be two disjoint subsets of Θ. It is not assumed that their union is Θ. The problem is to test sequentially H_1 versus H_2, where H_j is the hypothesis: $P = P_\theta$ for some $\theta \in \Theta_j$, $j = 1, 2$.

If the hypotheses H_j are simple, that is, $\Theta_j = \{\theta_j\}$, $j = 1, 2$, then Wald's Sequential Probability Ratio Test (SPRT) [23] computes the sequence of probability ratios

$$R_n = \prod_{i=1}^{n} \frac{p_2(Z_i)}{p_1(Z_i)}, \qquad n = 1, 2, \cdots, \tag{1.1}$$

Research supported in part by National Science Foundation Grant GP-23835.