SOME NEW RESULTS IN SEQUENTIAL ESTIMATION THEORY

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1. The setup of the problem; some general remarks

The problem of sequential estimation attracted some fresh interest recently. We can note, basically, two directions in the corresponding recent literature: asymptotic investigations (see for instance [1], [2], [3], [4]) and exact formulas for "small samples" (see [5], [6], [7], [8]). We shall give here an account of some recent results in both these directions.

In articles [1] to [4] a Bayesian approach to the sequential estimation of parameters is considered. Here we use another, non-Bayesian approach.

We can consider sufficiently large families of stochastic processes with independent increments (and discrete or continuous time). We shall always study scalar processes unless stipulated otherwise.

In the case of discrete time, we shall suppose it integer valued so that our process will be reduced to the repeated sample of a certain population with a distribution in a family $\mathscr{P}_\theta$ characterized by a density $f(x, \theta)$ with respect to the Lebesgue or the counting measure. The parameter θ will be always scalar, unless stipulated otherwise.

We shall consider here mostly the processes with discrete time, addressing ourselves, say, to the standard Poisson process only in Section 3.

However, many of the results expounded here can be transferred to the continuous time case.

In both cases we shall consider Markov stopping times τ (see [9] for the definition) and scalar statistics T_τ that are unbiased estimates of a scalar function $g(\theta)$ of the parameter θ

$$E_\theta(T_\tau) = g(\theta). \tag{1.1}$$

Moreover, we must choose among such unbiased estimates a statistic $\tilde{T}_\tau$ with the minimal variance $D_\theta(\tilde{T}_\tau)$ under the condition

$$E_\theta(\tau) \leqq n, \tag{1.2}$$

where n is a given number. From (1.2) it follows that $P_\theta(\tau < \infty) = 1$.

Note that if such a statistic exists, this statistic and the corresponding stopping rule may depend upon θ. Thus a statistic which is optimal in the above sense uniformly with respect to θ does not exist in general.