ON THE WIENER PROCESS APPROXIMATION TO BAYESIAN SEQUENTIAL TESTING PROBLEMS

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1. Introduction and summary

In 1959 Chernoff [7] initiated the study of the asymptotic theory of sequential Bayes tests as the cost of observation tends to zero. He dealt with the case of a finite parameter space. The definitive generalization of the line of attack initiated in that paper was given by Kiefer and Sacks in [13]. Their work as well as that of Chernoff, the intervening papers of Albert [1], Bessler [3], and Schwarz [19], and the subsequent work of the authors [4] used implicitly or explicitly the theory of large deviations and applied only to situations where hypothesis and alternative were separated or at least an indifference region was present.

In the meantime in 1961 Chernoff [8] began to study the problem of testing $H: \theta \leqq 0$ versus $K: \theta > 0$ on the basis of observation of a Wiener process with drift θ per unit time as an approximation to the discrete time normal observations problem. Having made the striking observation that study of the asymptotic behavior of the Bayes procedures for any normal prior was in this case equivalent to the study of the Bayes procedure with Lebesgue measure as prior and unit cost of observation, he reduced this problem for suitable loss functions to the solution of a free boundary problem for the heat equation. In subsequent work ([2], [9], [10] and [16]) the nature of this solution was investigated by Chernoff and others.

In this paper we are concerned with the problem of testing $H: \theta \leqq 0$ versus $K: \theta > 0$ by sampling sequentially from a member of one parameter exponential (Koopman-Darmois) family of distributions (see equation (3.1)) at cost c per observation. We will assume the simple zero-one loss structure in which an error in decision costs one unit while being right costs nothing.

[1]Prepared with the partial support of the Office of Naval Research, Contract NONR N00014-69-A-0200-1038.
[2]Prepared with the partial support of U.S. Public Health Grant GM-10525(07).
[3]This research was done while the author was visiting the University of California, Berkeley.