ON THE EXISTENCE OF PROPER BAYES MINIMAX ESTIMATORS OF THE MEAN OF A MULTIVARIATE NORMAL DISTRIBUTION

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1. Introduction

Consider the problem of estimating the mean of a multivariate normal distribution with convariance matrix the identity and sum of squared errors loss. In an earlier paper [5] the author showed that if the dimension p is 5 or greater, then proper Bayes minimax estimators do exist. We review this result briefly in Section 2.

The main purpose of the present paper is to show that for p equal to 3 or 4, there do not exist spherically symmetric proper Bayes minimax estimators. The author has been unable, thus far, to disprove the existence of a nonspherical proper Bayes minimax estimator for p equal 3 or 4. Of course, for $p = 1, 2$, the usual estimator $\overline{X}$ is unique minimax but not proper Bayes.

In Section 3 we derive bounds for the possible bias of a minimax estimator. This result should be of some interest independent of its use in proving the main result of the paper. Section 4 is devoted to the proof of the main result.

2. The case of five and higher dimensions

In [5] the author produced a class of estimators for $p \geqq 5$ which are proper Bayes minimax. This was done without loss of generality, in the case of a single observation. For completeness we briefly describe this result.

Let X be a p dimensional random vector distributed according to the multinormal distribution with mean θ and covariance matrix I.

The prior distribution on θ is given as follows: conditional on λ, where $0 < \lambda \leqq 1$, let the distribution of θ be multinormal with mean zero and covariance matrix $[(1 - \lambda)/\lambda]I$. The unconditional density of λ with respect to Lebesgue measure is given by $\lambda^{-a}/(1 - a)$ for any a where $0 \leqq a < 1$.

The Bayes estimator with respect to the above prior distribution on θ is given by

$$(1) \qquad \delta(X) = \left[1 - \left(\frac{p + 2 - 2a}{\|X\|^2} - \frac{2 \exp\{-\frac{1}{2}\|X\|^2\}}{\|X\|^2 \int_0^1 \lambda^{p/2 - a} \exp\{-\lambda\|X\|^2\}}\right)\right] X.$$