THE LIKELIHOOD RATIO TEST FOR THE MULTINOMIAL DISTRIBUTION

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1. Introduction and summary

Let $X^{(N)} = (X_1^{(N)}, \cdots, X_k^{(N)})$ be a random vector having a multinomial distribution with parameters N and $p = (p_1, \cdots, p_k)$,

$$(1.1) \qquad P(X^{(N)} = x|p) = \frac{N!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k},$$

where $x = (x_1, \cdots, x_k)$ is a vector with nonnegative integer components with sum N, and p is any point in the simplex

$$(1.2) \qquad \Omega = \left\{(y_1, \cdots, y_k) \middle| \sum_{i=1}^{k} y_i = 1, y_i \geqq 0 \text{ for } i = 1, \cdots, k\right\}.$$

By $Z^{(N)} = (Z_1^{(N)}, \cdots, Z_k^{(N)})$ we denote the random vector with components

$$(1.3) \qquad Z_i^{(N)} = \frac{X_i^{(N)}}{N}, \qquad i = 1, \cdots, k.$$

For $N = 1, 2, \cdots$, consider tests based on $Z^{(N)}$ for the hypothesis $H: p \in \Lambda_0$ against the alternative $K: p \in \Lambda_1$, where Λ_0 and Λ_1 are disjoint subsets of Ω and $\Lambda = \Lambda_0 \cup \Lambda_1$ may be a proper subset of Ω. It is assumed that the sizes α_N of the tests depend on N in such a way that $\alpha_N \to 0$ for $N \to \infty$. The likelihood ratio test based on $Z^{(N)}$ for H against K rejects H for large values of the statistic

$$(1.4) \qquad \inf_{p \in \Lambda_0} \sup_{\pi \in \Lambda} \sum_{i=1}^{k} Z_i^{(N)} \log \frac{\pi_i}{p_i},$$

possibly with randomization on the set where the statistic assumes its critical value.

In [2] W. Hoeffding considered a special case of this situation where $\Lambda = \Omega$, in which case the likelihood ratio statistic (1.4) reduces to

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