UPPER AND LOWER RISKS AND MINIMAX PROCEDURES

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The essential goal of R. A. Fisher's fiducial argument was to make posterior inferences about unknown parameters without resorting to a prior distribution. Over the past decade, there have been two major attempts at developing a statistical theory that would accomplish this convincingly. One of these efforts has been described in a series of publications by Fraser, the other in papers by Dempster. From the early work [4], [11], [12], [13], which was tied to a fiducial viewpoint, both authors developed statistical theories that were distinct from the fiducial argument, yet achieved the goal of non-Bayesian posterior inference [5], [6], [7], [8], [14], [15], [16].

Despite technical and other differences, the main ideas underlying this later work by Dempster and by Fraser appear to be similar. Fraser's papers, analyzing statistical models that possess a special kind of structure, arrive at "structural probability" distributions for the unknown parameters. Dempster's papers, dealing with less specialized models, derive "upper and lower probabilities" on the parameter space. Disregarding some technicalities, these upper and lower probabilities reduce to structural probabilities for the models considered by Fraser.

To this extent, upper and lower probabilities are a generalization of structural probabilities. However, there appear to be differences in interpretation. Fraser has given a frequency interpretation to structural probabilities in [11], [12] (but not in later work); this interpretation depends upon the special form of the statistical models in his theory, and does not apply to Dempster's theory. Dempster has provided no simple interpretation for upper and lower probabilities; he suggested in [7] that his theory might be "an acceptable idealization of intuitive inferential 'appreciations'." More recently, he has embedded his theory within a generalized Bayesian framework [9], [10]. The justification for the latter is unclear at present (see the discussion to [9]).

Lacking in both the Dempster and Fraser theories are systematic methods for dealing with estimation and hypothesis testing problems (or suitable analogues of such). A method of constructing tests was described by Fraser in [16], but no performance criteria were established. Dempster [5] defined upper and lower risks but did not pursue their application; the statistical meaning of these risks