ON PITMAN EFFICIENCY OF SOME TESTS OF SCALE FOR THE GAMMA DISTRIBUTION

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1. Summary

A comparison is made of several two sample rank tests for scale change in Gamma distributions. A test is studied, the exponential scores test, which offers greater Pitman efficiency than some standard tests (for example, the Wilcoxon) when the shape parameter γ is small.

2. Introduction

This paper considers a two sample testing problem which has arisen in connection with weather control (Neyman and Scott [6], section 5).

We let the random variable X represent the amount of rainfall during a day of nonzero precipitation under natural weather conditions, and let Y represent the rainfall in the same region during a day of rain in which the clouds have been seeded. It is assumed that X has a Gamma distribution and that the effect of seeding is multiplicative. The problem is to test for a positive seeding effect, using m observations on X and n observations on Y.

Formally, let $F_{\gamma,\delta}$ be the distribution on the positive real line with density

$$(2.1) \qquad f_{\gamma,\delta}(x) = \frac{\delta^{\gamma}}{\Gamma(\gamma)} x^{\gamma-1} e^{-\delta x},$$

where $\gamma > 0$, $\delta > 0$. The assumptions imply that when $F_{\gamma,\delta}$ is the distribution of X, then $F_{\gamma,\delta\xi}$ is the distribution of Y, for some $\xi > 0$. The problem becomes one of using samples $X_1, \cdots, X_m$ and $Y_1, \cdots, Y_n$ to test the hypothesis

$$(2.2) \qquad\qquad H:\xi \geqq 1$$

against the alternative

$$(2.3) \qquad\qquad K:\xi < 1.$$

Our primary goal is to determine a nonparametric test which will perform better than standard tests, such as the Wilcoxon, whose efficiency has been found to be unsatisfactory.