SENSITIVITY OF A BIRTH PROCESS TO CHANGES IN THE GENERATION TIME DISTRIBUTION

D. G. BURNETT-HALL and W. A. O'N. WAUGH
THE UNIVERSITY OF HULL

1. Introduction

This paper describes some studies on the age dependent binary branching process. These studies are partly theoretical and partly numerical. We shall first describe the model and then mention two problems which suggested this investigation.

The model is one that is sometimes used to represent the growth of cellular populations under favorable conditions, in which there are no deaths, and the life of each individual terminates by a process of fission giving rise to two new individuals. The life length of an individual is the period from its inception by binary fission of its parent to the instant of its own fission. This period varies randomly in the following sense. Let $G(t)$ be a distribution function (to be called the generation time distribution) which is such that $G(0-) = 0$. Suppose that at time 0 a single individual of age zero is present, and that its life length is L where $P\{L \leqq t\} = G(t)$. At the end of its life it is replaced by two individuals of age zero and their life lengths L_1, L_2 are independently distributed according to the same law; that is, $P\{L_1 \leqq t\} = P\{L_2 \leqq t\} = G(t)$. At the end of their lives these two individuals are each replaced by two newly born ones in the same way. Note that at any instant of time the probability of fission for each individual depends on its own age, but not on the number of others present, nor on absolute time. This model is a special case of the process that is described in Harris ([1], chapter VI). We shall summarize certain results given there (which we shall use), which are particularly concerned with the behavior of the process for large values of the time.

Let $Z(t)$ be the population size at time t.

Let ρ, which we shall call the "Malthusian parameter," be the unique real positive root of the equation

$$\int_0^\infty e^{-\rho t}\, dG(t) = \frac{1}{2}. \tag{1}$$

Let

$$n_1 = \left[4\rho \int_0^\infty te^{-\rho t}\, dG(t)\right]^{-1}. \tag{2}$$