MODELS FOR ANTIBODY ATTACHMENT TO VIRUS AND BACTERIOPHAGE

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1. Introduction

Several interesting mathematical problems concerned with partition into classes, and surface covering are suggested by the physical mechanisms and geometry of antibody attachment to virus particles. This paper outlines some of the recent work done in this area by Yassky [7], myself [2], [3], Moran and Fazekas de St. Groth [5] and Gilbert [4], adds a model and other extensions of my own, and concludes with a suggestion for further investigations. I shall endeavor throughout to hold the mathematical argument at a simple level, and emphasize the model building aspect of the work, in the hope that virologists may be tempted to use and perhaps verify experimentally some of the models put forward.

Let us consider at any time $t \geqq 0$, a nutrient medium (either in the laboratory, or within a living animal) in which there exist a fixed number N of particles of a virus V; suppose that at time $t = 0$, there are $x_0 > N$ antibodies released into this medium. We may expect the antibodies to attach themselves progressively in some random fashion to the viruses, both types of particles being subject to Brownian motion. If each virus particle permits a maximum of s attachments, then at any time $t \geqq 0$, the N virus will be divided into $s + 1$ classes consisting of $n_0(t)$, $n_1(t)$, $\cdots$, $n_s(t)$ particles with, respectively, $0, 1, \cdots, s$ antibodies attached to them; there will remain $x(t) = x_0 - \sum_{i=1}^{s} i n_i(t)$ unattached antibody particles. The $\{n_i(t)\}$ constitute a class partition of the virus particles, which varies in time t. We may, for simplicity, in some cases, approximate the integer valued random variables $0 \leqq n_i(t) \leqq N$, and $0 \leqq x(t) \leqq x_0$ by analogous functions differentiable in t; then, as we shall see, a deterministic approximation to the random evolution of the $\{n_i(t)\}$ and $x(t)$ can be found. It is also possible to obtain a stochastic approximation to the integer valued $\{n_i(t)\}$ using the previous deterministic approximation for $x(t)$.

While such results may indicate the number of antibody attachments to the virus, they do not alone provide adequate information as to its loss of infectivity. Two cases arise, however, in which fuller information can be obtained. These are the cases when

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