ON THE GENERAL STOCHASTIC EPIDEMIC

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1. Introduction

The purpose of this paper is to survey some recent results obtained in the solution of the model for the general stochastic epidemic, which was originally proposed by Bartlett [3]. Various aspects of the general epidemic, particularly in the stationary state, have previously been considered in detail by Bailey [1], [2], Whittle [9], Foster [4], and Kendall [7], among others. Around October 1964, Siskind at University College, London, and I at Michigan State University, Lansing, independently arrived at explicit time dependent solutions for this model; our complementary results, which differ in various details, have appeared in *Biometrika* (1965; Vol. 52, Parts 3 and 4). What I shall attempt to outline here is an improved method of solution for the general stochastic epidemic; this is, I believe, simpler than any so far proposed, and provides greater insight into the structure of the model. The same approach can also be used to attack recurrent epidemic processes for which a solution has been sketched (cf. Gani, [6]).

The stochastic epidemic model considered is that for which at time $t \geqq 0$, there are in circulation in a closed population of size $n + a$ with $n, a \geqq 1$,

(i) $0 \leqq r \leqq n$ uninfected susceptibles,
(ii) $0 \leqq s \leqq n + a - r$ infectives,

the remaining $n + a - r - s \geqq 0$ individuals having been removed through immunity or death. At time $t = 0$, the population is known to consist of n susceptibles and a infectives.

Let the probabilities of possible transitions in the interval $(t, t + \delta t)$ be

$$\begin{aligned} \Pr\{r, s \rightarrow r - 1, s + 1\} &= rs\delta t + o(\delta t), \\ \Pr\{r, s \rightarrow \quad r, \quad s - 1\} &= \rho s\delta t + o(\delta t), \end{aligned} \tag{1.1}$$

where for convenience the usual infection parameter β is set equal to 1 and ρ denotes the (relative) removal parameter. The process $\{r, s\}$ is Markovian, and the transition probabilities of r susceptibles and s infectives at time $t \geqq 0$,

$$p_{rs}(t) = \Pr\{r, s \text{ at time } t | n, a \text{ at time } 0\} \tag{1.2}$$

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