BRANCHING PROCESSES AND THE THEORY OF EPIDEMICS

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1. Introduction

In the present paper we shall discuss the extinction problem for certain branching processes. Our main purpose is to study those branching processes which can serve as models of epidemics; that is, spreads of an infectious disease. The phenomenon of epidemics is fairly complex, and all models necessarily have to be based on a certain compromise. This compromise consists of taking into account some of the (presumably important) factors governing the spread of the disease at the cost of neglecting others. Our main idealization will consist of assuming such mechanisms of infection which yield a branching process. Using informal language, it means that all infectives present in the population at a given time infect the susceptibles independently of each other. More precisely, if there are k infectives in the nth generation of the epidemics, then the distribution of the next $(n + 1)$st generation can be represented as the distribution of a sum of k independent, identically distributed random variables, with a specified distribution. These random variables represent the "progeny" of k infectives of nth generation.

It is debatable whether the above assumption is justified in the sense that models based on it provide an adequate description of reality. The main objection is that when the population gets "saturated" with infectives, the assumption of independence becomes violated. We shall not attempt to defend our models; we only want to show that it is possible to fit within the framework of branching processes some of the factors governing the spread of the disease; such as, effects of vaccination and other preventive methods, variable lengths of the period of incubation and infectiousness, effects due to random movements of infectives within the habitat, and also, to some extent, the effects due to the exhaustion of susceptibles. We do obtain meaningful results within our models; whether or not these results bear any relation to reality lies beyond our present interest.

In section 2 we consider a simple case of a Galton-Watson process, and we show that even in this relatively simple case one can take into consideration the variable length of the period of incubation and infectiousness. In section 3 we generalize the model of section 2 to include the random movements of individuals.

Finally, in section 4 we deal with generalized branching processes, in an