ESTIMATING THE TRAJECTORY OF A POPULATION

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1. Introduction

The population trajectory with which this paper is concerned is the changes in numbers of people which would result from certain birth and death rates, specific by age and sex, when these are applied to a given initial age distribution. Considering either sex, let the probability that a person aged x to $x + 4$ years at last birthday will survive for five years be ${}_5L_{x+5}/{}_5L_x$; the age specific fertility rates for age x to $x + 4$ be m_x; the number of individuals alive at the time t be ${}_5K_x^{(t)}$. (In general the superscript on the upper right in parentheses will refer to time, that on the lower right to the initial age of the interval, that on the lower left to the length of the interval.) This paper will estimate the path of ${}_5K_x^{(t)}$ subject to two restrictions: (a) that the age specific rates of birth and death are constant; and (b) that their application is without any random variation, which is to say the argument will be entirely in terms of expected values. The extension of the method to rates varying in time, and for probabilistic as well as for deterministic models, is important, and one hopes that it will attract the attention of the good minds needed to cope with it.

The conditions of birth and death set up in the preceding paragraph enable us to show the relation between the population at time $t + 1$ and that at time t, where t is in units of five years, as a set of linear, first order, homogeneous, difference equations with constant coefficients

$$
\begin{aligned}
\frac{{}_5L_0}{2\ell_0}\{[{}_5K_{15}^{(t)} + {}_5K_{15}^{(t+1)}]m_{15} + [{}_5K_{20}^{(t)} + {}_5K_{20}^{(t+1)}]m_{20} \qquad & \\
+ \cdots + [{}_5K_{40}^{(t)} + {}_5K_{40}^{(t+1)}]m_{40}\} &= {}_5K_0^{(t+1)} \\
\frac{{}_5L_5}{{}_5L_0}\,{}_5K_0^{(t)} &= {}_5K_5^{(t+1)} \\
\vdots \qquad & \qquad \vdots \\
\frac{{}_5L_{85}}{{}_5L_{80}}\,{}_5K_{80}^{(t)} &= {}_5K_{85}^{(t+1)}.
\end{aligned}
\tag{1.1}
$$

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