MATHEMATICAL MODELS FOR NEURAL NETWORKS

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1. Introduction

The study of neural network models contributes indirectly to the study of the way in which brains work. The neurons which make up a brain have a number of known physiological properties; the brain operating as a whole has other known properties. By investigating the overall behavior of assemblies of cells which have some, but not all, of the properties of neurons and comparing this behavior with that of a brain it should be possible to decide the role and the relative importance of the properties that neurons have. In this paper examples of various models are given and others may be found from the references. Much of the recent work on networks has been influenced by ideas relating to the design of high speed digital computers; the question that is asked is, essentially, how a stimulus applied to an assembly at rest is transmitted or transformed by it. This is a static approach. A dynamic approach would require us to regard a brain as having its own way of behaving, even in isolation, and to ask how this behavior is modified by a stream of stimuli from its environment. A very simple model of this sort is proposed.

2. Definition of a network

From the point of view of someone making a mathematical model, a neural network is an assembly of objects (cells) with the following properties:

(i) at a given time, a cell can be active (firing) or inactive;

(ii) cells are connected by paths; a pair of cells A, B may be unconnected, or connected by any number of paths; these paths are directional;

(iii) if there is a path $\overrightarrow{AB}$, the firing of A at some time may contribute to the firing of B at a later time; A may be described as an input cell for B;

(iv) a cell fires either as the result of an external stimulus or because of the firing of its input cells.

Various details about these properties must be specified in the model. The inactive state referred to in (i) is usually classified into two states (a) used, for some period after the cell has fired, during which it cannot fire again (a recovery time or refractory period), (b) sensitive, when the cell can be reactivated. In (iii) the time for a signal to go from A to B (the synaptic delay) must