SOME INEQUALITIES FOR RELIABILITY FUNCTIONS

Z. W. BIRNBAUM
UNIVERSITY OF WASHINGTON
and
J. D. ESARY
BOEING SCIENTIFIC RESEARCH LABORATORIES

1. Basic concepts and definitions

1.1. With the advent of very complex engineering designs such as those of high-speed computers or supersonic aircraft, it has become increasingly important to study the relationship between the functioning and failure of single components and the performance of the entire system and, in particular, to be able to make quantitative statements about the probability that the system will perform according to specifications. It is the aim of this paper to present some inequalities for this probability.

1.2. We shall assume that there are only two states possible for every component of a system, as well as for the system itself: either it functions or it fails. When the system consists of n components, we shall ascribe to each of them a binary variable which will indicate its state

$$x_i = \begin{cases} 1 & \text{when the } i\text{-th component functions,} \\ 0 & \text{when the } i\text{-th component fails} \end{cases} \tag{1.2.1}$$

for $i = 1, 2, \cdots, n$. Similarly, we ascribe to the entire system a binary indicator variable

$$u = \begin{cases} 1 & \text{when the system functions,} \\ 0 & \text{when the system fails.} \end{cases} \tag{1.2.2}$$

When the design of a system is known, then the states of all n components (that is, the values of $x_1, x_2, \cdots, x_n$) determine the state of the system, that is the value of u so that

$$u = \phi(x_1, x_2, \cdots, x_n) \tag{1.2.3}$$

where ϕ is a function assuming the values 0 or 1. This function ϕ will be called the *structure function* of the system. The indicator variable x_i will sometimes be referred to as "component x_i" and ϕ will sometimes be called "structure ϕ." The n-tuple of 0's or 1's

$$(x_1, x_2, \cdots, x_n) = \underline{x} \tag{1.2.4}$$

will be called the *vector of component states* or, in short, the "state vector." It