ON SOME STOCHASTIC PROBLEMS OF RELIABILITY THEORY

YU. K. BELYAEV, B. V. GNEDENKO and A. D. SOLOVIEV
MOSCOW UNIVERSITY

1. Introduction

The new direction in scientific investigations, associated with modern tendencies in engineering and called "reliability theory," has imposed enormous demands on the theory of probability and mathematical statistics. Naturally, in the present short paper we are forced to restrict our selection to just a small number of problems.

Without the ideas and concepts of the theory of probability, even the fundamental concepts of reliability theory cannot be clearly defined. Therefore, the theory of probability is not only a computational apparatus, but also a methodological basis for reliability theory. An inadequate perception of the nature of those phenomena which must be encountered in problems of reliability theory often leads engineers to certain misunderstandings. An attempt for greater clarity naturally obliges us to turn to the elaboration of general initial concepts.

We understand the reliability of an object to be the capacity to retain the properties determining its quality unscathed. All possible states of the object which are equivalent from the viewpoint of its reliability, will be combined in the class x. The set of all possible classes x generates a phase space of the states of the object E.

For example, if the object consists of n units each of which may be either in the in-service or out-of-service state, the phase space E is generated by points $(\epsilon_1, \epsilon_2, \cdots, \epsilon_n)$ whose coordinates may take only two values: $\epsilon_i = 0$ if the i-th unit is in service, and $\epsilon_i = 1$ if the i-th block is not in service. Under the assumption that the out-of-service blocks have been repaired and work serviceably during random time intervals with an exponential distribution, the state of the object is described entirely by the fact that the object is at some point $(\epsilon_1, \epsilon_2, \cdots, \epsilon_n)$ of the phase space E at the time t. If the out-of-service units are subject to repair which lasts a time distributed according to $F(t) \neq 1 - e^{-\mu t}$, we must also indicate the time τ_i during which the failing unit with number i is repaired, in order to describe the state of the system. Here we assume that each unit goes to repair immediately after going out of service. Therefore, in this case we should examine a more complex phase space E consisting of points of the form $(\epsilon_1, \tau_1, \cdots, \epsilon_n, \tau_n)$. If $\epsilon_i = 0$, then τ_i is also assumed to be zero. If $\epsilon_1 = 1$, then $0 < \tau_i < \infty$.

Various changes associated with wear (in mechanical systems), or with aging