THE DETERMINISTIC STOCHASTIC TRANSITION IN CONTROL PROCESSES AND THE USE OF MAXIMUM AND INTEGRAL TRANSFORMS

P. WHITTLE
UNIVERSITY OF MANCHESTER

1. Introduction

Consider the "minimum transform" $\Phi(y)$ of a function $F(x)$ defined by

$$\Phi(y) = \min_x [F(x) - xy]. \tag{1.1}$$

Then, under certain conditions on F, the essential one being that of convexity, the inverse relation is simply

$$F(x) = \max_y [\Phi(y) + xy], \tag{1.2}$$

that is, F is the "maximum transform" of Φ. We shall refer to transforms of either type generically as "maximum transforms."

In this paper we shall show that use of the transform leads to a very natural treatment of certain control problems.

The pair of relations (1.1), (1.2) is strikingly analogous in form to a Fourier integral transformation and its inversion. The analogy is more than fortuitous. Consider a pair of functions F, Φ linked via the reciprocal Fourier relations

$$\begin{aligned} \exp\left[\frac{\Phi(y)}{c}\right] &= \text{const.} \int \exp\left[\frac{F(x) - xy}{c}\right] dx \\ \exp\left[\frac{F(x)}{c}\right] &= \text{const.} \int \exp\left[\frac{\Phi(y) + xy}{c}\right] dy, \end{aligned} \tag{1.3}$$

where the integration contours are taken appropriately, one of them at least leaving the real axis. We suppose that c is a small constant. As c tends towards zero it is plausible that the integrals in (1.3) can be asymptotically evaluated by steepest descents, so that the relations (1.3) between F and Φ will reduce to relations (1.1) and (1.2).

The significance of this transition will emerge from our heuristic examination of some rather special cases. We shall consider some control problems in which there is a random disturbance whose variance is measured by c. In some cases these can be solved by use of Fourier transforms. As c tends to zero and the