SEQUENTIAL DECISIONS IN THE CONTROL OF A SPACESHIP

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1. Introduction and summary

Imagine a spaceship travelling towards a certain planet with predetermined speed, in a direction which will bring it close to the target after a known period of time. Observations on the position of the target, relative to the present course, are made continuously and lead to a gradually improving prediction of the eventual miss distance. On the other hand, the fuel available in the spaceship for making minor changes in the direction of motion, is gradually losing its effectiveness. This is because the final change of position caused by a small velocity imposed perpendicular to the present motion, is roughly proportional to the remaining time. Thus we have a control problem which is essentially one of compromise between the extremes of using the fuel early and perhaps in the wrong way, because of poor information; or waiting too long for more precise information, so that the fuel becomes ineffective.

The statistical decision problem considered here actually arises from a simplified formulation of the above question, but one which contains its main features. We suppose first that the motion of the spaceship relative to its target is confined to a fixed plane with the target as origin. The horizontal component of velocity is fixed as unity, so that the time coordinate $\tau \leq 0$, also represents the horizontal distance to be travelled before the target is passed. It is enough to represent the vertical components of position and velocity together by μ, the height at which the present line of motion meets the axis $\tau = 0$. However, μ is unknown, and must be estimated continuously by observing a certain stochastic process $\{W(\tau); \tau \leq 0\}$ whose mean drift is μ per unit time.

A second fiction, which will be maintained throughout the present paper, is that an infinite quantity of fuel is available for adjusting the vertical velocity, and hence μ, at a fixed price c per unit change of velocity. Thus at any time τ, an instantaneous velocity increment Δ, costing $c\Delta$, will change the unknown quantity μ by a known amount $\Delta|\tau|$. The problem is to find a control procedure which minimizes the sum of all fuel costs together with a cost associated with the final miss. For the most part, we shall assume that this terminal cost is given by $\frac{1}{2}k\mu^2$. Because of the symmetry of this function, the direction of any control

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