SPORADIC RANDOM FUNCTIONS AND CONDITIONAL SPECTRAL ANALYSIS: SELF-SIMILAR EXAMPLES AND LIMITS

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1. Introduction

The definition of sporadic random functions has arisen out of a pessimistic evaluation of the suitability of ordinary random functions as models of certain "turbulencelike" chance phenomena. Now that one has evaluated a number of spectra of turbulence, one must indeed admit that their sampling behavior is often not at all as predicted by the developments of the Wiener-Khinchin second order theory. True, given two frequency intervals (λ_1, λ_2) and (λ_3, λ_4), the ratio between the energies in these intervals rapidly tends to a limit, as was expected. But the energies within each of the intervals (λ_1, λ_2) or (λ_3, λ_4) continue to fluctuate wildly, however large the sample may be. Another puzzling fact: some turbulencelike phenomena appear to have an infinite energy in low frequencies, a syndrome often colorfully referred to as an "infrared catastrophe."

The reason the concept of stationary random function must be generalized may be further explained as follows. Many physical time series X are "intermittent," that is, alternate between periods of activity, and quiescent "intermissions" during which X is constant and may even vanish. Moreover, and this is the crucial point, some series are quiescent *most* of the time. Using the intuitive *physical* meaning of the phrase, "almost sure event," such a series would appear to satisfy the following properties, which will entitle it to be called "sporadic": X is almost surely constant in any prescribed finite span (t', t''), but X "almost surely" varies sometime. It would be very convenient if each of the above occurrences of the term "almost sure" could also be interpreted in the usual mathematical terms. Unfortunately, one has so rigged the theory of random functions, that the above two requirements are incompatible ([8], pp. 51 and 70).

There is, however, a simple way of generalizing the concept of a stationary random function, so as to accommodate the sporadically varying series $X(t)$. It suffices to amend in two ways the classical Kolmogorov's probability space triplet $(\Omega, \mathcal{A}, \mu)$: (a) the measure μ is assumed unbounded though sigma-finite; and (b) a family $\mathcal{B}$ of conditioning events B is added, where $0 < \mu(B) < \infty$. It is further agreed that, henceforth, the only well-posed questions will be those relative to conditional probabilities $\Pr\{A|B\}$ where $B \in \mathcal{B}$.

The first part of the present paper is devoted to the preliminary task of