THE SPECTRAL ANALYSIS OF LINE PROCESSES

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1. The specification of line processes

In recent papers [2], [3], I have discussed the spectral analysis of point processes in one or more dimensions, showing that the degenerate character of such processes does not prevent spectral analysis techniques, already familiar with continuous processes being adapted to such processes. The question arises, somewhat analogously as in the case of spectral or other distribution functions themselves, whether other forms of degeneracy will be encountered in practice; and, if so, what procedures are possible. One class of process which does arise in various contexts is what I have termed a *line process* ([2], p. 295) in which the points of a point process are replaced, in two or more dimensions, by lines. The example given referred to a number of vehicles on a road, treated for simplicity as points in a one-dimensional continuum, and thus at any instant as a point process. If the points are considered at two instants of time we have a bivariate point process, but if the points are plotted continuously over time as another coordinate the process will consist of a number of lines. This example makes two things clear. First, the specification of the process is partly optional, for the same process is either a point process (in a coordinate x, say) developing in time, or a "static" two-dimensional line process in x and the time coordinate t. Such alternative representations are not exhaustive, for (as in dynamics) the velocity u could also be included if convenient as an additional coordinate, though of course this is not necessary, as u is always derivable in the other specifications. Second, the lines in the line process need not be straight, as when the vehicles are accelerating. Indeed, in any general mathematical specification the lines might not even possess tangents at any point, as in a collection of Brownian particles. We shall, however, for definiteness assume that derivatives exist, as in our example. Moreover, as in the case of point processes, only particular classes of processes can be statistically analyzed by standard techniques. In the case of point processes, spectral analysis requires stationarity (or the equivalent property in more than one dimension). When discussing the spectral analysis of line processes, we shall not only assume an appropriate stationarity property, but shall also for simplicity consider processes consisting merely of *straight* lines, though not necessarily of infinite extent. Such a process in two dimensions is sometimes useful as an idealized representation of the fibers in a sheet of paper.