RANDOM PACKING DENSITY

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1. Introduction

Random packing and random space-filling problems have received attention in recent articles in the literature. Investigation of seemingly disparate topics has produced the analyses and the results that are published, and there may be other settings in which related work has been underway that is not available in print. Writings on this subject appear in a wide variety of journals, and it is not difficult to miss pertinent published papers.

In one dimension, this topic usually bears the label "parking problem," and this has been treated by several authors. Here, the packing, or space filling, is achieved by automobiles of unit length which are parked at random, one at a time, in the unfilled intervals remaining on the line until space for one car is no longer available. The center of the interval representing the length of the automobile is assumed to follow a uniform distribution over the unfilled portions of the line. The mean, variance, and distribution of the proportion of the line filled by cars in this manner, as the length of the line becomes infinite, is the object of the analysis. Rényi's paper [13] is the first on this subject and it is followed by Ney [10], and most recently by Mannion [9] and Dvoretzky and Robbins [7]. We will return to these in later sections.

Related one-dimensional problems for the discrete situation are analyzed by Jackson and Montroll [8] and Page [11]. Page considers pairs of adjacent points selected at random from n points on a straight line, one pair at a time, such that neither point of a selected pair is allowed then to form a pair with its other neighbor. The process of selection is repeated until the only points remaining are isolated from one another by intervening pairs already selected. The proportion of isolated points as n approaches infinity is a parameter of interest in some physical models and we shall return to this study. Jackson and Montroll investigated the number of possible configurations for n points on a line, the total number of vacancies in all these configurations, and then the average proportion of vacancies in a configuration. In both papers, the actual physical problem is two-dimensional, but this is very difficult to manage; therefore, resort is made to one-dimensional analogues which are interesting in their own right and may suggest approaches and approximations for the two-dimensional model. Both sets of studies, namely the discrete and continuous models, differ from a large class of studies on counter problems and renewal theory models because there is a dependence of later random fillings on those previously registered.