HARNESSES

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1. Long-range misorientation in the crystalline structure of metals

Sometime in 1955 or 1956 at the Atomic Energy Research Establishment at Harwell, Professor A. H. Cottrell set me a problem on long-range misorientation in the crystalline structure of metals. This made me think about multidimensional martingales, which I propose to call harnesses. The whole subject seems largely unexplored, and I have only some sketchy and tentative remarks to make; but I resurrect this material in the hope that others will bring it to fruition.

When we analyze the micro-structure of a large lump of metal, we find two major features; first, the metal consists of an assemblage of imperfect metal crystals, called grains; and second, the grains consist of small contiguous domains (called subgrains) of irregular size and shape, in each of which the atoms are packed on a perfect crystalline lattice. Each subgrain has a unit vector specifying the spatial orientation of its lattice. The vectors of adjacent subgrains subtend small irregular angles, one with the next, say random angles with some specified distribution. It is these angles between subgrains that are the manifestation of the imperfection of the crystal structure in the grains. Except in special circumstances (which do not concern us here) the orientations of neighboring grains are quite independent; but those of neighboring subgrains are not independent because the change of orientation from one to the next is small. A grain contains a large number of subgrains. One might suppose that the small irregularities between neighboring subgrains would cumulate, so that the long-range misorientation (namely the angle between the vectors of two subgrains within the same grain but many subgrains apart) would increase with increasing distance between the subgrains and so ultimately become quite large. Yet observation shows this is not to be the case: the long-range misorientation seems to be of the same order of magnitude as the short-range misorientation between adjacent subgrains. Indeed, it is this smallness of the long-range misorientation which is the manifestation of the (albeit imperfect) overall crystalline coherency of the grain. The general orientation of the structure can only change by large amounts at the so-called "large-angle grain boundaries" between one grain and the next. Much the same sort of effect seems to occur if one tries to distort at random a sheet of paper by crumpling it up. The normal to the surface of the paper has a more or less constant direction (or else varies quite smoothly and not at all randomly) except at the