EXISTENCE OF PHASE TRANSITIONS IN MODELS OF A LATTICE GAS

R. L. DOBRUSHIN
UNIVERSITY OF MOSCOW

1. Introduction

It is proved here that at sufficiently low temperatures, a phase transition occurs in the model of a lattice gas with pairwise interaction of the particles, if a constraint, meaning roughly that the negative part of the potential in some sense "outweighs" its positive part, is imposed on the interaction potential; or if the potential is nonzero, nonpositive, and decreases sufficiently rapidly at infinity. The proof is based on a further development of the method introduced independently by the author in [1], [2] for the proof of the existence of a phase transition in the Ising model of a lattice gas, and by Griffiths [3] for the solution of a similar problem. Using the same method, Berezin and Sinai [4] proved the existence of a phase transition in models of a lattice gas with a nonpositive finite potential, which is negative in the segment $[0, R]$.

All the constructions presented below are carried out analogously for lattices of any dimensionality greater than one (as is known, there are no phase transitions in one-dimensional lattices). For greater clarity, we carry out the reasoning for two-dimensional lattices (the generalization to higher dimensions is described in detail in [2]).

Let V_ℓ be a square with side ℓ in a two-dimensional square lattice, that is, the set of points $X = (x_1, x_2)$, $x_i = 1, 2, \cdots, \ell$; $i = 1, 2$. We shall call the subset $a = (X_1, \cdots, X_N)$ of N elements of V_ℓ the *arrangement* of N particles in the square V_ℓ. We denote the set of all such arrangements by $\mathcal{V}_{N,\ell}$. For clarity, we shall often interpret V_ℓ as a square piece of graph paper with unit square cells by assigning to the point of the lattice a cell whose center is this point. The arrangement a is thereby interpreted as a way of choosing N of the ℓ^2 cells in V_ℓ, which are declared *filled*, while the rest, including the cells outside V_ℓ, are *empty*. The *potential* will be a function $U(Y)$ defined in the set R of all integer, two-dimensional vectors Y, except zero, and depending only on the length $|Y|$ of the vector Y. (The results extend almost without change to the case in which $U(Y)$ can depend on the direction of Y.) The number

$$Z(N, \ell, T) = \sum_{a \in \mathcal{V}_{N,\ell}} \exp\left\{-\frac{1}{T} \sum_{i<j} U(X_i - X_j)\right\} \tag{1.1}$$

is called the statistical sum. The constant $T > 0$ is the gas *temperature*. Suppose that