TRANSITION PROBABILITY OPERATORS

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1. Summary

First, a few remarks are made on invariant probability measures of a transition probability operator. The notion of irreducibility is introduced for a transition operator acting on the continuous functions on a compact space, and implications of this assumption are examined. Finally, the unitary and isometric parts of the operator are isolated and interpreted in terms of the behavior of iterates of the operator.

2. Preliminary remarks

Let us consider a space Ω with a Borel field of subsets $\mathcal{B}$. We shall call $P(x, B)$, $x \in \Omega$, $B \in \mathcal{B}$, a transition probability function if $P(x, \cdot)$ is a probability measure on $\mathcal{B}$ for each $x \in \Omega$ and $P(\cdot, B)$ is a $\mathcal{B}$-measurable function for each $B \in \mathcal{B}$. Higher order transition probability functions can then be introduced recursively starting with $P(\cdot, \cdot)$:

$$(1)\qquad \begin{aligned} P_1(x, B) &= P(x, B), \\ P_{n+1}(x, B) &= \int P_1(x, dy)P_n(y, B), \qquad n = 1, 2, \cdots, \quad x \in \Omega, \quad B \in \mathcal{B}. \end{aligned}$$

It can easily be seen that the functions $P_n(\cdot, \cdot)$ probability measures on $\mathcal{B}$ for each $x \in \Omega$ and measurable in x for each $B \in \mathcal{B}$. Further, a small argument shows that

$$(2)\qquad P_{n+m}(x, B) = \int P_n(x, dy)P_m(y, B), \qquad n, m = 1, 2, \cdots$$

There is an operator T taking bounded measurable functions into bounded measurable functions induced by $P(\cdot, \cdot)$:

$$(3)\qquad (Tf)(x) = \int P(x, dy)f(y).$$

The operator T is positive in that $Tf \geq 0$ if $f \geq 0$; by $f \geq 0$ we mean that $f(x) \geq 0$ for all $x \in \Omega$. Also, T maps the function one onto itself. A convenient

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