STRONG MIXING PROPERTIES OF MARKOV CHAINS WITH INFINITE INVARIANT MEASURE

KLAUS KRICKEBERG
UNIVERSITY OF HEIDELBERG

1. Introduction

It is common knowledge that the existence of weakly wandering sets for ergodic transformations T in spaces of infinite measure excludes the existence of strongly mixing transformations in such spaces if this concept is defined by requiring the "mixing equation" (2.2) below to be true, with some sequence ρ_n, for all measurable sets A and B of finite measure. However, Hopf's famous example of a transformation in the plane ([19], p. 67) shows that (2.2) may hold for all bounded sets A and B whose boundary has measure 0. This and the desire to put the so-called "individual" or "strong" ratio limit property of Markov chains into the general framework of measure preserving transformations might motivate the present attempt to treat mixing in topological measure spaces, and to restrict the attention to almost everywhere continuous transformations.

The second section describes the measure spaces and the class of their admissible isomorphisms. These isomorphisms, too, are required to be almost everywhere continuous, and thus leave invariant the concept of a mixing and, more generally, of a quasi-mixing transformation as defined here. What is involved is, essentially, a particular kind of weak convergence of functions of two sets which are sigma-finite measures in each variable.

One of the basic tools is the construction of mixing transformations in Euclidean spaces from the shift in the sample space of Markov chains via an isomorphism between these two measure spaces; this isomorphism is given in section 3. The next section describes the relation between the strong ratio limit property and the quasi-mixing and mixing property of the shift. Section 5 treats some examples, and the final one contains, without proofs, category theorems which can be obtained by exploiting these methods systematically.

The present paper owes much to oral and written discussions with F. Papanghelou, much more, indeed, than will be apparent from the references given below. In particular, he drew my attention to the papers [14], [16], [18], and [21].

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