CLASSIFICATION OF STATES FOR OPERATORS

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1. Summary

We characterize the sets of positive states and null states for nonsingular Markov processes and, more generally, for positive contractions in L_1. The set P of positive states is an invariant set and carries all finite invariant measures which are absolutely continuous with respect to a given measure μ, the initial distribution. The Cesaro averages of the "probabilities of being in B at time n" converge to a positive limit for any subset B of P with $\mu(B) > 0$. The set N of null states is a countable union of sets X_i with the property that the Cesaro averages of the "probabilities of being in X_i" tend to 0 for each X_i. We further generalize Hopf's decomposition of the state space into a conservative and dissipative part by introducing monotonically decreasing weights, obtaining the positive part P as a special "weighted conservative part" with divergent sum of weights. As an application we derive an ergodic theorem with appropriate weighted averages under conditions which do not imply the usual ergodic theorem (corollary 2).

Different characterizations of the decomposition into P and N have been described by Mrs. Dowker [7] (for point mappings) and by Neveu [20]. (See also Neveu's paper of this Berkeley Symposium. I noticed the decomposition independently, but later than Neveu. Also A. Hajian and Y. Ito have some related (so far unpublished) results, which overlap with Neveu's present paper and are based on his paper [20].) I am indebted to Professors D. Freedman, Y. Ito, and W. Pruitt for some references.

2. Introduction

Let $(X, \mathfrak{F}, \mu)$ be a measure space with $\mu(X) = 1$. All sets and functions introduced are assumed to be measurable. Sets as well as functions are identified if they coincide almost everywhere. Let T be a positive contraction in $L_1 = L_1(X, \mathfrak{F}, \mu)$, that is, a linear operator in L_1 with $Tf \geq 0$ for all $0 \leq f \in L_1$, and with $||T|| = \sup_{||f||=1} ||Tf|| \leq 1$. By the Radon-Nikodym theorem, L_1 is isomorphic to the Banach space Φ of all signed measures φ which are absolutely con-

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