# CONSERVATIVE POSITIVE CONTRACTIONS IN $L^1$

ARSHAG B. HAJIAN[1] and YUJI ITO[2]
NORTHEASTERN UNIVERSITY and BROWN UNIVERSITY

## 1. Introduction

Let $(X, \mathcal{B}, m)$ be a $\sigma$-finite measure space, and let $T$ be a positive contraction defined on $L^1(X, \mathcal{B}, m)$. It was shown by Hopf [9] that the space $X$ can be decomposed into two disjoint subsets with respect to $T$, the conservative part and the dissipative part. Recently, Neveu [12] remarked that one can single out a particular subset, called the strongly conservative part, from the conservative part. In what follows, we show (theorem 2) that if the positive contraction $T$ is conservative, then the complement of the strongly conservative part can be decomposed further into at most a countable number of disjoint subsets, each of which is characterized by a certain infinite sequence of positive integers. As Neveu has remarked, the strongly conservative part is characterized as the maximal set carrying an element $f$ in $L^1(X, \mathcal{B}, m)$ which is left invariant by $T$ (see theorem 1 below).

The problem of determining the existence of a strictly positive element which is left invariant by $T$ has received considerable attention in recent years in connection with the invariant measure problem for measurable transformations and Markov processes. Various necessary and sufficient conditions for the existence of such an element $f$ (though stated in terms of the existence of a finite, invariant, and equivalent measure) have been obtained by Hopf [8], Dowker [3], [4], Calderón [1], Hajian and Kakutani [6], and Sucheston [13] for the case of an operator $T$ which arises from a measurable transformation; and by Ito [10] and Hajian and Ito [5] for the case of a $T$ which arises from a Markov process. The methods and results of the last two papers cited above can be generalized further without any modification. In fact, Neveu [12] proves some of these assertions for the general case of a positive contraction $T$ operating on $L^1(X, \mathcal{B}, m)$ using much simpler and more elegant arguments.

In Hajian and Ito [5] it was shown by means of a trivial counter-example that the case of Markov processes in general is not quite the same as for the case of invertible measurable transformations. However, assuming that the operator is conservative, most of the annoying minor difficulties disappear, and the theory generalizes smoothly. In the second part of this paper we show

[1] Research supported in part by an NSF grant, GP-4574.
[2] Research supported in part by an NSF grant, GP-3422.