A GEOMETRIC CONSTRUCTION OF MEASURE PRESERVING TRANSFORMATIONS

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1. Introduction

Our main purpose is to give an extension of a construction which we have obtained in [3]. A consequence of the discrete spectrum theorem is that transformations having discrete spectrum have a square root if and only if -1 is absent from the spectrum. One of the important problems in ergodic theory is to investigate to what extent the implications of the discrete spectrum theorem remain in the general case. Along these lines, Halmos [10] has asked whether every transformation with continuous spectrum has a square root. We remark that the question is raised (see [8]) for one-to-one transformations since otherwise the discrete spectrum theorem as well as the consequence it mentioned fail to hold. Note furthermore that the spectral theorem holds only for one-to-one transformations, and therefore we can expect the spectrum to determine the properties of the transformation in this case only.

In [3] an example is obtained which answers the question of Halmos to which we have referred. Here we extend the method to give an example of a one-to-one ergodic and measure preserving transformation which has continuous spectrum, is not strongly mixing, and which has no roots of any order. The fact that the transformation is not strongly mixing is of some interest, as in view of the fact that the shift transformations are strongly mixing.

The paper is divided into three sections. In the first we give a simple example due to Kakutani and von Neumann. In the second section we formulate and prove a general decomposition theorem for one-to-one measure preserving transformations. The sufficient part of this theorem is a summary and codification of ideas common to constructions given by several authors ([6], [9], [12], and [13]). The example given in the first section is the simplest as well as the earliest which exhibits the main features of the theorem. Our main intention in giving the decomposition theorem is to fix ideas for the construction of the counter-example which is the principal purpose of the paper. This construction is given in the third section and is independent of the first two.

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