UNIQUENESS OF STATIONARY MEASURES FOR BRANCHING PROCESSES AND APPLICATIONS

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1. Outline of the problem

A one-dimensional Markov branching process may be characterized as follows. An organism, at the end of its lifetime (of fixed duration), produces a random number ξ of offspring with probability distribution

$$\Pr\{\xi = k\} = a_k, \qquad k = 0, 1, 2, \cdots, \tag{1}$$

where as usual

$$a_k \geq 0, \qquad \sum_{k=0}^{\infty} a_k = 1. \tag{2}$$

All offspring act independently with the same fixed lifetime and the same distribution of progeny. The population size $X(n)$ at the n-th generation is a temporally homogeneous Markov chain whose transition probability matrix is

$$P_{ij} = \Pr\{X(n+1) = j | X(n) = i\} = \Pr\{\xi_1 + \xi_2 + \cdots + \xi_i = j\}, \tag{3}$$

where ξ's are independent observations of a random variable with the probability law (1). An equivalent way to express (3) is through its generating function, which is simply

$$\sum_{j=0}^{\infty} P_{ij} s^j = [f(s)]^i, \qquad i = 0, 1, \cdots, \tag{4}$$

where $f(s) = \sum_{k=0}^{\infty} a_k s^k$.

It is a familiar fact that the n step transition probability matrix $P_{ij}^{(n)} = \Pr\{X(n) = j | X(0) = i\}$ possesses the generating function

$$\sum_{j=0}^{\infty} P_{ij}^{(n)} s^j = [f_n(s)]^i, \tag{5}$$

where

$$f_n(s) = f_{n-1}(f(s)), \qquad f_0(s) = s, \tag{6}$$

is the n-th functional iterate of $f(s)$.

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