A LIMIT THEOREM FOR INDEPENDENT RANDOM VARIABLES

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The starting point of this paper is the question, what happens to the distribution of the sum of a large number of independent, identically distributed, integer valued, random variables or equivalently, what happens to a measure on the group of integers when convoluted by itself a large number of times? It is known that the probability of being at a fixed integer tends to a limit and that this limit is 0. Therefore, the finer information about what the distribution looks like is obtained by looking at the ratio of the probability of being at one fixed integer to the probability of being at another fixed integer. Such a theorem was proved by Chung and Erdös in [1].

There are two natural directions for generalizing this theorem. One generalizes to a Markov process and the other to convoluting measures on a more general group.

A generalization to Markov chains is given by Kingman and Orey in [3]. Another generalization is given by Jain in [2] for a fairly general Markov process, but the price for the generality is that the theorem is about the ratio of the expected number of visits to a set up to time n instead of the probability of being there at time n.

In this paper we generalize to convoluting on more general groups and prove a theorem in the case where the group is the line. The method used is a modification of the one used by Chung and Erdös. This method gives the same theorem for Euclidean space, and if we analyze the proof, we see that we use very little that is specific to the line, and hence we could get a theorem for a general locally compact abelian group. (Our assumption of mean 0 is used only in obtaining lemma 1, and hence in all cases when we have lemma 1, we have a general theorem.) We could do the same for the time ratio, thus generalizing theorem 4 of [1].

Charles Stone recently gave another proof of the main theorem of this paper in [4]. His method seems to give more information in the case of Euclidean space but does not seem to go over to more general groups.

THEOREM. *Let X_n be a sequence of independent, identically distributed, real-valued random variables with either mean 0 or with the integral of the positive and negative parts both infinite. Assume also that the values X takes are not all part of an arithmetic progression. Let $S_n = \sum_{i=1}^{n} X_i$. Let J_1 and J_2 be two intervals of the same length. Then*