A NOTE ON MARKOV SEMIGROUPS WHICH ARE COMPACT FOR SOME BUT NOT ALL $t > 0$

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1. Introduction

In this note $\{P_t\colon t \geq 0\}$ will be a strongly continuous semigroup of transition operators on ℓ and R_λ will be the corresponding resolvent operator for $\lambda > 0$.

The following three statements are correct.

(i) If, for some $t > 0$, P_t is quasi-compact, then it is quasi-compact for all $t > 0$.

(ii) [(iii)] If, for some $\lambda > 0$, λR_λ is compact [quasi-compact], then it is compact [quasi-compact] for all $\lambda > 0$.

Statements (i) and (ii) are very easy to prove, and (iii) is established in each of the accompanying papers by D. Williams and J. G. Basterfield. This note shows that the fourth similar assertion is false by giving an example of a Markov semigroup for which P_t is compact if $t > 1$, but not compact if $0 < t < 1$.

2. The example

The states are labelled 0 and (m, n) for $n = 1, 2, \cdots$ and $m = 1, 2, \cdots, n$. Here 0 is an absorbing state and the (m, n)-th state feeds into the $(m - 1, n)$-th state at rate $n(m > 1)$, $(1, n)$ feeds into 0 also at rate n.

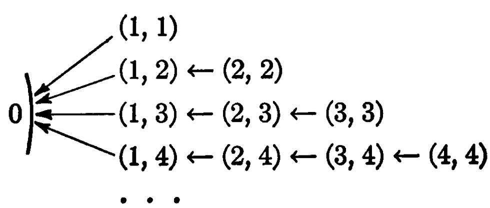

FIGURE 1

Thus $p_{(n,n),0}(t)$ is the distribution function of the sum of n independent negative exponential random variables, each with mean $1/n$; hence, if $t < 1$, then $p_{(n,n),0}(t) \to 0$ as $n \to \infty$ and if $t > 1$, then $p_{(n,n),0}(t) \to 1$. It is also clear that $p_{(m,n),0}(t) \geq p_{(n,n),0}(t)$ for $1 \leq m \leq n$ and for all t, and that for $j \geq N$,