ON MARKOV GROUPS

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1. Introduction

Let $\{P_t\colon 0 \le t < \infty\}$ be a strongly continuous one-parameter semigroup of transition operators on the Banach space ℓ_1 of absolutely convergent series, reducing to the identity at $t = 0$, with matrix representation

$$(P_t x)_j = \sum_i x_i p_{ij}(t), \qquad (j = 1, 2, \cdots), \tag{1}$$

so that in $\{p_{ij}(\cdot)\colon i, j = 1, 2, \cdots\}$ we have a standard family of Markov transition functions on $[0, \infty)$ in the terminology of K. L. Chung [1]. In this and in the succeeding papers [5], [6] we shall be interested in three loosely related questions:

(i) the analyticity or otherwise of the functions $p_{ij}(\cdot)$;

(ii) the identification of quasi-analytic classes of such functions;

(iii) the possibility of extending the Markov semigroup $\{P_t\colon 0 \le t < \infty\}$ to a strongly continuous *group* $\{P_t\colon -\infty < t < \infty\}$ of bounded linear operators on ℓ_1.

The present paper is concerned with the last of these three topics, and consists largely of conjectures and scraps of evidence about them. Some further evidence will be found in the accompanying paper by Miss J. M. O. Speakman [7]. If our remarks lead others to solve the problems posed, we shall be delighted.

2. Property (U) and property (G)

It will be helpful to make the following definitions.

DEFINITION 1. *A Markov semigroup* $\{P_t\colon 0 \le t < \infty\}$ *will be said to have property* (U) *when any one of the following equivalent conditions is satisfied:*

$(U1)$: $p_{ii}(t) \to 1$ *as* $t \to 0$, *uniformly with regard to* i;
$(U2)$: $\|P_t - I\| \to 0$ *as* $t \to 0$;
$(U3)$: $P_t = \exp(At)$, *where* A *is a bounded operator;*
$(U4)$: $\sup q_i < \infty$, *where* $q_i = -p'_{ii}(0)$.

The equivalence of $(U1)$ and $(U2)$ is due to the fact that

$$\|P_t - I\| = 2 \sup_i (1 - p_{ii}(t)). \tag{2}$$

The equivalence of $(U2)$ and $(U3)$ is a standard result in the theory of such semigroups ([4], theorem 9.6.1). The equivalence of $(U123)$ and $(U4)$ follows from the fact that

$$1 - p_{ii}(t) \le 1 - \exp(-q_i t) \le q_i t, \tag{3}$$