SOME THEOREMS CONCERNING RESOLVENTS OVER LOCALLY COMPACT SPACES

HIROSHI KUNITA
NAGOYA UNIVERSITY

AND

TAKESI WATANABE*
THE INSTITUTE FOR ADVANCED STUDY and KYUSHU UNIVERSITY

1. Introduction and preliminaries

This article is concerned with two problems on resolvents defined over locally compact separable Hausdorff spaces; the generation of a strong Markov process from the given substochastic resolvent (sections 3 and 4) and the representation of excessive measures by means of minimal (or extreme) excessive measures (sections 5 to 11). These problems are closely connected, and our approach to them is based on the results of D. Ray [19] concerning resolvents over compact metric spaces (see section 2) and the metric completion of the original locally compact space with respect to the uniformity generated by a certain family of bounded continuous functions. Two types of the metric completion will be introduced; the completion of F. Knight [14] (in a specific way) in section 3 and the completion of R. S. Martin in section 6.

In the rest of this section we will give some basic definitions as well as a brief description of the first problem. Let $(E, \mathcal{B})$ be a measurable space and $\mathcal{A}$, the σ-field formed by all universally measurable sets, tnat is, sets which, for each finite measure μ over $\mathcal{B}$, differ by at most a set of (μ) measure zero from a set of $\mathcal{B}$. A (nonnegative) real-valued function $R_\alpha(x, A)$ defined for $\alpha > 0$, $x \in E$ and $A \in \mathcal{B}$ is said to be a *resolvent* if the following conditions are satisfied. (R_1) For each $\alpha > 0$ and $x \in E$, $R_\alpha(x, \cdot)$ is a measure over $(E, \mathcal{B})$. (R_2) For each $\alpha > 0$ and $A \in \mathcal{B}$, $R_\alpha(\cdot, A)$ is measurable $(\mathcal{A})$. (R_3) The resolvent equation

$$R_\alpha(x, A) - R_\beta(x, A) + (\alpha - \beta) \int R_\alpha(x, dy) R_\beta(y, A) = 0, \qquad \alpha, \beta > 0 \tag{1.1}$$

is satisfied. The unspecified integral always means the integral over the whole space E. (R_4) The substochastic condition

$$\alpha R_\alpha(x, E) \leq 1 \qquad \text{for every} \quad \alpha > 0 \quad \text{and} \quad x \in E \tag{1.2}$$

* Work supported by the National Science Foundation through the Institute for Advanced Study and the University of Illinois.