APPLICATION OF ADDITIVE FUNCTIONALS TO THE BOUNDARY PROBLEM OF MARKOV PROCESSES (LÉVY'S SYSTEM OF U-PROCESSES)

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1. Introduction

Under the name of boundary problem for Markov processes, we shall consider the problem of finding all Markov processes whose behavior, before they reach the boundary, is the same as that of given minimal processes (cf. Feller [2]). In this paper, we shall characterize those processes by their U-processes (on the boundary) and certain auxiliary factors (cf. Sato [8]). The precise formulations and the summary of the paper are given in sections 2 and 3. The author wishes to express his gratitude to Professors M. Nagasawa, K. Sato, and T. Ueno for their kind discussions and advice, and especially to Professor K. Sato who read the paper and suggested many improvements.

2. Assumptions and notations

The space S is a compact space with metric r and $S^* = S \cup \{\partial\}$ where ∂ is an isolated (extra) point; D is a fixed open set in S such that $S = \bar{D}$ and V is the boundary of D. As sample paths on the space $S^* = S \cup \{\partial\}$ or $V^* = V \cup \{\partial\}$, we consider paths which are right continuous, have left limits, and stay at ∂ after they reach ∂. The path is denoted by w and $x_t(w) \in S^*$ (or $\xi_t(w) \in V^*$) is the value of w at t. We shall set $x_\infty(w) = \partial$, $\sigma_E = \inf \{t > 0\colon x_t(w) \in E\}$, and $\zeta = \inf \{t > 0\colon x_t(w) \in D\}$ with $\inf \phi = \infty$. A Markov process defined on the space of the above sample paths is called a Hunt process if it satisfies the conditions (P.1), (P.2), (P.3), and (P.4) in [5]. Roughly speaking, a Hunt process is a right-continuous and quasi-left-continuous strong Markov process. When referring to "subsets of S," we shall mean only topological Borel subsets of S. For $E \subset S$, $B(E)$ is the set of all bounded measurable functions on E, and $B^+(E)$ (resp. $C(E)$) means the subset of $B(S)$ consisting of the functions which are nonnegative (resp. continuous). Sometimes, we consider f in $B(E)$ as a function on δ^*, setting $f(x) = 0$, $x \notin E$. For E, $F \subset S$, $K(x, A)$, $(x \in E, A \subset F)$ is called a kernel on $E \times F$ if $K(\cdot, A)$ is Borel measur-