THE MARTIN BOUNDARY OF RECURRENT RANDOM WALKS ON COUNTABLE GROUPS

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1. Introduction

In [16] and [17] Spitzer investigated recurrent random walks on the integers or integral points in the plane from a potential theoretical point of view. Spitzer proved the existence of a potential kernel $A(x, y)$ and found its asymptotic behavior as well as limits of hitting probabilities of finite sets. These results were extended by Spitzer and the author to random walks on countable abelian groups in [13], and the present paper considers recurrent random walks on arbitrary countable groups.

The author is obliged to Professor Spitzer for several discussions on the subject matter of this paper.

An outline of the results follows. The existence of the potential kernel and its simplest properties still hold in the general case, but the asymptotic behavior of the potential kernel was successfully studied only for special groups. More specifically, let $\mathfrak{G}$ be a countable, infinite group with identity element e. A random walk, abbreviated as r.w. in the sequel, on $\mathfrak{G}$ is a (homogeneous) Markov chain $X_0, X_1, \cdots$ with state space $\mathfrak{G}$ and transition probabilities

$$(1.1) \qquad P(x, y) = P\{X_{n+1} = y | X_n = x\} = p(x^{-1}y), \qquad x, y \in \mathfrak{G}$$

where

$$(1.2) \qquad p(z) \geq 0, \qquad \sum_{z \in \mathfrak{G}} p(z) = 1.$$

(The letters x, y, z always denote elements of $\mathfrak{G}$ and x^{-1} is the inverse of x in $\mathfrak{G}$, and xy the product of x and y in $\mathfrak{G}$.) In other words, X_{n+1} is obtained from X_n by right multiplication with the random group element $X_n^{-1}X_{n+1}$ which has the distribution

$$(1.3) \qquad P\{X_n^{-1}X_{n+1} = z\} = p(z).$$

As usual,

$$(1.4) \qquad P_k(x, y) = P\{X_{n+k} = y | X_n = x\} = P_k(e, x^{-1}y)$$

is the x, y entry of the k-th power of P, if $k \geq 1$ and

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