ACCESSIBLE TERMINAL TIMES

R. M. BLUMENTHAL and R. K. GETOOR
UNIVERSITY OF WASHINGTON

1. Introduction

Let $X = (\Omega, \mathfrak{M}, P^x, X_t, \theta_t)$ be a Hunt process having a locally compact space E with a countable base as state space. We refer the reader to the expository paper ([4] or [1], pp. 133–134), for all concepts and notations which are not explicitly mentioned in the present paper.

A stopping time T for the process X is called *accessible* if for each initial measure μ on E there is a nondecreasing sequence $\{T_n\}$ of stopping times such that P^μ almost surely, $T_n \rightarrow T$ and $T_n < T$ for all n on $\{T > 0\}$. Meyer [7] has proved the remarkable result that a stopping time T is accessible if and only if the path $t \rightarrow X_t(\omega)$ is continuous at $T(\omega)$ almost surely on $\{T < \infty\}$. We will say that a stopping time T is *thin* if $P^x(T > 0) = 1$ for all x in E. As usual, an analytic subset A of E is thin if $P^x(T_A > 0) = 1$ for all x in E, where $T_A = \inf \{t > 0 \colon X_t \in A\}$ is the hitting time of A. These definitions are consistent since clearly A is thin if and only if T_A is thin. Finally a stopping time T is called a *terminal time* if for each t

$$T = t + T \circ \theta_t, \qquad \text{almost surely on} \quad \{T > t\}. \tag{1.1}$$

If A is an analytic subset of E, then T_A is a terminal time and the phrase "almost surely" may even be dropped from statement (1.1).

Let us now assume that X satisfies Hunt's hypothesis (F). (See [5], [6], or [1], pp. 133–134.) It then follows from proposition 18.5 of [5] that T_A is an accessible terminal time whenever A is a thin analytic subset of E. Moreover, it is clear that $T_A = \infty$ on $\{T_A \geq \zeta\}$ if $A \subset E$. The main result of this paper is the following converse of the above statement.

THEOREM 1. *Assume X satisfies hypothesis (F). If T is a thin accessible terminal time with the property that $P^x[\zeta \leq T < \infty] = 0$ for all x, then there exists a thin Borel set $B \subset E$ such that $T = T_B$ almost surely.*

The proof of theorem 1 is given in section 2; then in section 3 we give some applications of theorem 1 to the structure of natural additive functionals.

Consider the following process: the state space $E = L \cup L_1 \cup L_2$ is the following subset of the Euclidean plane, $L = \{(x, y) \colon x \leq 0, y = 0\}$ is the nonpositive x-axis, L_1 is the segment joining the points $(0, 1)$ and $(1, 0)$, whereas L_2 is the segment joining $(0, -1)$ and $(1, 0)$. The process consists of translation to the right at unit speed until $(0, 0)$ is reached. The point $(0, 0)$ is a holding point

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