AN ALGEBRAIC TREATMENT OF FLUCTUATIONS OF SUMS OF RANDOM VARIABLES

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1. Introduction

Let $X_1, \cdots, X_n$ be n symmetrically dependent random variables defined on a probability space $(\Omega, \mathfrak{F}, P)$. By symmetrically dependent it is meant that the joint distribution of $X_1, \cdots, X_n$ is invariant under permutations of the random variables.

The present paper is concerned with relations between the distributions of several functions of the variables $X_1, \cdots, X_n$.

The notion of symmetrically dependent random variables is closely connected to the concept of interchangeable random variables introduced by de Finetti [1]. De Finetti, however, assumes the existence of an infinite sequence of random variables, such that each finite subsequence is a set of symmetrically dependent random variables. It is easy to show by examples that there exists for each n a set of n random variables, which are symmetrically dependent, but such that it is not possible to extend the set to an infinite sequence of interchangeable random variables.

For interchangeable random variables it has been proved [1] that the distribution is a mixture with positive weights of distributions of independent, identically distributed random variables. This result does not hold in general for symmetrically dependent random variables as the following example shows.

Let X_1, X_2 be symmetrically dependent random variables such that $P(X_1 = 1, X_2 = 0) = P(X_1 = 0, X_2 = 1) = \frac{1}{2}$; then no mixture with positive weights of distributions of pairs of independent, identically distributed random variables yields the distribution of X_1 and X_2.

2. Symbolic convolutions and their relations

The symmetrically dependent random variables $X_1, \cdots, X_n$ will be called basic random variables. All other random variables will be defined in terms of these basic ones.

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