ON THE ARITHMETICAL PROPERTIES OF CERTAIN ENTIRE CHARACTERISTIC FUNCTIONS

EUGENE LUKACS
THE CATHOLIC UNIVERSITY OF AMERICA

1. Introduction

Let $f(t)$ be a characteristic function and suppose that the corresponding distribution function has a finite second moment. It is then known that

$$T_1 f(t) = \frac{f'(t) - f'(0)}{t f''(0)} \tag{1.1a}$$

and

$$T_2 f(t) = \frac{f''(t)}{f''(0)} \tag{1.1b}$$

are also characteristic functions. These operators may be applied to analytic characteristic functions repeatedly and yield again analytic characteristic functions. The present study is motivated by the wish to investigate the arithmetical properties of the two families of characteristic functions which are obtained if the operator T_2^k or $T_1 T_2^{k-1}$ is applied to the characteristic function $f(t) = \exp(-t^2/2)$.

It is, however, convenient to investigate a more general class of characteristic functions, namely the family of entire characteristic functions of order two which have only a finite number of zeros. (In view of a theorem of Marcinkiewicz (see [4], p. 156), this family is identical with the entire characteristic functions of finite order which have only a finite number of zeros.) In the following, we denote this class by $\mathfrak{G}_2$. In section 2, we give some formulae concerning Hermite polynomials and discuss the construction of characteristic functions which belong to $\mathfrak{G}_2$. Section 3 deals with factorization problems of characteristic functions of $\mathfrak{G}_2$.

2. The class $\mathfrak{G}_2$

It is well known (see [4], p. 134) that the zeros of analytic characteristic functions cannot have an arbitrary location but are subject to the following restrictions:

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