GENERAL BRANCHING PROCESSES WITH CONTINUOUS TIME PARAMETER

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1. Introduction

The notion of a general branching process was introduced in ([1] chapter III). The general branching process is a Markov branching process whose state space Ω consists of all nonnegative integral-valued measures concentrated on finite subsets of a given space X. In [1], the discrete time-parameter case is studied in detail.

In the present paper we shall be interested in the continuous time-parameter case, and we shall restrict ourselves to the purely discontinuous Feller type. This restriction, not allowing diffusion of individual particles, is natural for some basic spaces X and generally for those processes where the types of particles change by fission only. In [1] references are given to papers studying general branching processes with a kind of diffusion of individual particles and with a simple fission. The present paper does not include these examples as special cases; on the other hand, it studies the purely discontinuous case in full generality. The axiomatic treatment presents certain existence problems which are solved in section 2. In section 3 we shall provide necessary and sufficient conditions for the degeneration of the process. We may expect that the general case could be studied in a similar way if Feller's pure-discontinuity condition were replaced by a kind of mixed-type condition.

In the whole paper we shall use, with few exceptions, the same notation as in [1]; in particular, X will denote the space of types of particles. We shall assume that X is a σ-compact metric space (that is, a denumerable union of compact subsets), and we shall denote by $\mathfrak{X}$ the corresponding σ-algebra of Borel sets in X. By Ω, we shall denote the set of all nonnegative measures ω on $\mathfrak{X}$, which are concentrated on finite subsets of X and assume integral values. Each element $\omega \in \Omega$ may be characterized by a double vector $(x_1, n_1; \cdots; x_k, n_k)$ where $\{x_1, \cdots, x_k\}$ is the finite subset of X on which ω is concentrated (that is, $\omega(\Omega - \{x_1, \cdots, x_k\}) = 0$) and $n_i = \omega(\{x_i\})$. According to the definition, n_i is a nonnegative integer. If we denote by $\bar{x}$ the measure concentrated at the point $x \in X$ and which assumes there the value 1, we may express the relation between ω and the corresponding double vector $(x_1, n_1; \cdots; x_k, n_k)$ by $\omega = \sum_{i=1}^{k} n_i \bar{x}_i$.

We shall denote by $\mathcal{Y}$ the Kolmogorov σ-algebra in Ω, that is, the least σ-algebra containing all cylinder sets $\{\omega \in \Omega: \omega(\{x\}) = n\}$, $x \in X$, n an integer. The set of all bounded $\mathfrak{X}$-measurable functions on X will be denoted by $\mathfrak{F}$, and