INTEGRATION OF CORRESPONDENCES

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1. Introduction

The traditional economic concept of a set of agents, each of which cannot influence the outcome of their collective activity but certain coalitions of which can influence that outcome, has recently received its proper mathematical formulation by means of measure theory. After J. W. Milnor and L. S. Shapley [32] had considered in 1961 a game with a measure space of players (see also [39], [41], [40], [13], [33]), in an article [2] published in 1964, R. J. Aumann showed how two basic concepts for an economy, namely the set of competitive allocations and the core, coincide when the set of consumers is an atomless positive finite measure space. Another solution of this equivalence problem based on Lyapunov's theorem [28] was then given by K. Vind [42]. In the light of this result, measure theory indeed appears as the natural context in which to study economic competition. (The concept of a continuum of agents has also been used in economic theory for different purposes by R. G. D. Allen and A. L. Bowley ([1], pp. 140–141) and H. S. Houthakker [23].)

Now, given a finite set A of agents and a real Banach space S (the commodity space), a standard operation in the analysis of economic equilibrium consists of associating with every element a of A a nonempty subset $\varphi(a)$ of S, that is, of defining a correspondence φ from A to S in Bourbaki's [9] terminology, and of taking the sum $\sum_{a \in A} \varphi(a) = \{z \in S | z = \sum_{a \in A} f(a)$ for some function f from A to S such that, for every $a \in A, f(a) \in \varphi(a)\}$.

In the new measure-theoretic context, the set A of agents is an arbitrary set; the set $\mathcal{A}$ of coalitions if a σ-field of subsets of A; a countably additive nonnegative real-valued function ν is defined on $\mathcal{A}$ with the interpretation that, for a coalition $E \in \mathcal{A}$, $\nu(E)$ is the fraction of the totality of agents contained in E. In this context the sum $\sum_{a \in A} \varphi(a)$ must be replaced by the integral $\int_A \varphi \, d\nu$ of the correspondence φ. Thus it becomes necessary to define this integral and to study its properties, in order to be able to reformulate the theory of economic equilibrium. In [3], Aumann has made to this problem a fundamental contribution which this article proposes to extend in several directions. The first extension aims at replacing his assumption that the set of agents is an analytic set by the assumption that it is a measurable space. From the viewpoint of economic interpretation, this generalization is important, for the identification of

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