CAPACITY AND BALAYAGE FOR DECREASING SETS

M. BRELOT
UNIVERSITY OF PARIS

1. Introduction

A general concept of capacity (extracted by Bourbaki from Choquet's theory [8]) which we shall call general real capacity, is a real function (finite or not) $\mathcal{C}(e)$ of a set e in a Hausdorff space, satisfying the following conditions:

(i) $\mathcal{C}(e)$ is increasing; (ii) for any increasing sequence e_n, $\sup \mathcal{C}(e_n) = \mathcal{C}(\bigcup e_n)$; (iii) for any decreasing sequence of compact sets e_n, $\inf \mathcal{C}(e_n) = \mathcal{C}(\bigcap e_n)$.

Taking the value of $\mathcal{C}$ in a complete lattice [9] (with a greatest and a smallest element), these conditions may be considered as defining a "general capacity" and we shall meet interesting examples where it is a function of a variable in a suitable lattice of functions.

There has been no general study of the limit of a capacity for a decreasing sequence of noncompact sets. We shall develop and complete a lecture in the seminar on potential theory (November 1964) and a note [5] by giving some examples from balayage theory (and first from the most classical capacity) where the previous limit is made precise. Among these capacities, one will be a function, in the family (ordered by $\leq$ everywhere) of the superharmonic functions. The famous capacitability theorem for the real capacities holds also for this capacity. We shall even consider directed decreasing, and also increasing, sets of sets, and similar set functions which are not capacities but are given by balayage theory, in an axiomatic frame of potential theory, as well as in the classical case. (General research of Doob is in course and has given or inspired the extensions to directed sets, as it will be mentioned in the text.) After the lecture I developed here, I became aware of connected axiomatic discussions by Fuglede (see *C. R. Acad. Sci. Paris*, October 1965.) For this research and these results, the fine closed sets (that is, closed according to the fine topology) play an essential role and the basic tool is a theorem by Choquet on thinness (lemma 1).

Finally, an application will be made to get a proof of a theorem by Getoor (on a smallest fine closed support of a measure) that was first obtained by probability theory, then proved and generalized by Choquet [11] in an axiomatic way. Another application will generalize some results by using, as Doob proposed, the fine upper semicontinuous functions.

We shall work in pure potential theory without giving any probabilistic interpretation and we shall be able to shorten the redaction by referring to a recent paper [4] containing connected or similar concepts, tools or proofs