OUTLINE OF SOME TOPICS IN LINEAR EXTRAPOLATION OF STATIONARY RANDOM PROCESSES

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1. Introduction

We shall consider the frequently met problem of linear extrapolation of the stationary random processes $x(s)$, $-\infty < s < \infty$, with $Ex(s) = 0$. The problem consists of finding that linear functional $\hat{x}(t; \tau)$, $\tau > 0$, of the values $x(s)$ for $s \leq t$ (extrapolation according to the entire past of the process) or for $t - T \leq s \leq t$ (extrapolation of a process given on a finite interval) which would give the best approximation to the random variable $x(t + \tau)$. "Best" here is intended in the sense of least-squares; that is, it is required of the functional $\hat{x}(t; \tau)$ that the mean-square prediction error

$$\sigma^2(\tau) = E|x(t + \tau) - \hat{x}(t; \tau)|^2 \tag{1}$$

takes on its minimum value.

A. N. Kolmogorov [1], [2] initiated the theory of linear least-squares extrapolation of stationary processes. This theory was developed further by M. G. Kreĭn [3], N. Wiener [4], K. Karhunen [5], and others. At present, it has achieved a significant degree of completion (see, for example, Doob [6], chapter XII, or Rozanov [7]). We may formulate the general solution of this problem in the following way.

Let us start from the spectral representation of the stationary stochastic process in the form

$$x(s) = \int_{-\infty}^{\infty} e^{is\lambda} \, dZ(\lambda) \tag{2}$$

where $Z(\lambda)$ is the stochastic measure on the $-\infty < \lambda < \infty$ axis. This measure is connected to the spectral function $F(\lambda)$ of the process $x(s)$ by the relationship

$$E\left\{\int_S dZ(\lambda) \cdot \int_{S_1} \overline{dZ(\lambda)}\right\} = \int_{S \cap S_1} dF(\lambda), \tag{3}$$

where the bar above the symbol signifies the complex conjugate. If $F'(\lambda)$ is zero on a set of nonzero Lebesgue measure, or if $F'(\lambda)$ is not zero almost everywhere but

$$\int_{-\infty}^{\infty} \frac{|\log F'(\lambda)|}{1 + \lambda^2} \, d\lambda = \infty, \tag{4}$$