SOME PREDICTION PROBLEMS FOR STRICTLY STATIONARY PROCESSES

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1. Introduction

A strictly stationary process $\{x_t\}$ $(-\infty < t < \infty)$ is one whose distributions remain the same as time passes; that is, the multivariate distribution of the random variables $x_{t_1+h}, x_{t_2+h}, \cdots, x_{t_n+h}$ is independent of h. Here $t_1, t_2, \cdots, t_n$ is any finite set of parameter values. Throughout this paper we shall assume that the expectation $E|x_t|$ is finite, $Ex_t = 0$ and $\lim_{h\to 0} E|x_{t+h} - x_t| = 0$. Strictly stationary processes satisfying these additional conditions will be called shortly *stationary*. Moreover, random variables which are equal with probability 1 will be treated here as identical.

Let $[x_t]$ denote the linear space spanned by all random variables x_t $(-\infty < t < \infty)$ and closed with respect to the mean convergence. Of course, $[x_t]$ becomes a Banach space under the norm $\|x\| = E|x|$. Moreover,

$$\|x\| \leq \|x + y\| \qquad \text{if } x \text{ and } y \text{ are independent} \tag{1.1}$$

(see [8], p. 263). It is well known that to each stationary process $\{x_t\}$ there corresponds a unique one-parameter strongly continuous group $\{T_t\}$ of linear operators in $[x_t]$ preserving the probability distribution and such that $x_t = T_t x_0$ (see [1], chapter XI, section 1). Conversely, each such group $\{T_t\}$ in conjunction with a random variable y with $Ey = 0$ defines a stationary process $x_t = T_t y$.

Let $[x_t: t \leq a]$ be the subspace of $[x_t]$ spanned by all random variables x_t with $t \leq a$. We say that a stationary process $\{x_t\}$ *admits a prediction*, if there exists a linear operator A_0 from $[x_t]$ onto $[x_t: t \leq 0]$ such that

(i) $A_0 x = x$ whenever $x \in [x_t: t \leq 0]$,

(ii) if for every $y \in [x_t: t \leq 0]$ the random variables x and y are independent, then $A_0 x = 0$,

(iii) for every $x \in [x_t]$ and $y \in [x_t: t \leq 0]$ the random variables $x - A_0 x$ and y are independent.

The random variable $A_0 x$ can be regarded as a linear prediction of x based on the full past of the process $\{x_t\}$ up to time $t = 0$. An optimality criterion is given by (iii). In what follows the operator A_0 will be called *a predictor* based on the past of the process up to time $t = 0$. The conditions (i), (ii), and (iii) determine the predictor A_0 uniquely.