A CONTRIBUTION TO THE MULTIPLICITY THEORY OF STOCHASTIC PROCESSES

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1. Introduction

In a paper [1] read before the Fourth Berkeley Symposium in 1960, I communicated the elements of a theory of spectral multiplicity for stochastic processes. A related theory was given about the same time by Hida [5]. Since then, I have developed the theory in some subsequent papers [2]–[4], the most recent of which contains the text of a lecture given at the Seventh All-Soviet Conference of Probability and Mathematical Statistics in Tbilisi 1963. Further important work in the field has been made by Kallianpur and Mandrekar [6]–[8].

Many interesting problems arising in connection with this theory are still unsolved. The object of this paper is to offer a small contribution to the investigation of one of these problems.

We shall begin by giving in section 2 a brief survey of the results of multiplicity theory so far known for the simplest case of one-dimensional processes. For proofs and further developments we refer to the papers quoted above. A major unsolved problem will be discussed in section 3, whereas section 4 is concerned with some aspects of the well-known particular class of stationary processes, which are relevant for our purpose. Finally, section 5 is concerned with the construction of a class of examples which may be useful in the further study of the problem stated in section 3.

2. Spectral multiplicity of stochastic processes

Consider a stochastic process $x(t)$, where $x(t)$ is a complex-valued random variable defined on a fixed probability space, while t is a real-valued parameter. In general we shall allow t to take any real values, and shall only occasionally consider the case when t is restricted to the integers. We shall always assume that the relations

$$(2.1) \qquad Ex(t) = 0, \qquad E|x(t)|^2 < \infty$$

are satisfied for all t.

We denote by $H(x)$ the Hilbert space spanned in a well-known way by the random variables $x(t)$ for all t, while $H(x, t)$ is the subspace of $H(x)$ spanned