GENERALIZED UNIFORM COMPLEX MEASURES IN THE HILBERTIAN METRIC SPACE WITH THEIR APPLICATION TO THE FEYNMAN INTEGRAL

Dedicated to Professor Charles Loewner

KIYOSI ITÔ
KYOTO UNIVERSITY and STANFORD UNIVERSITY

1. Introduction and summary

As we pointed out in the Fourth Berkeley Symposium [4], an infinite dimensional version of the complex measure in the k-space E_k,

$$F_k(dx) = \lambda_k(dx)/(\sqrt{2\pi hi})^k, \qquad \lambda_k = \text{Lebesgue measure} \tag{1}$$

is useful for a mathematical formulation of the Feynman integral [1]; h is a positive constant which is supposed to indicate the Planck constant in its application to quantum mechanics and $\sqrt{z}$, $(z \neq 0)$ denotes the branch for which $-\pi/2 < \arg \sqrt{z} < \pi/2$ throughout this paper. Since neither λ_k nor $(\sqrt{2\pi hi})^k$ has any meaning when $k = \infty$, we cannot directly extend this measure to the infinite dimensional space E_∞ (Hilbert space). Therefore, we shall consider a linear functional $F_k(f)$ induced by the measure F_k in (1):

$$F_k(t) = \int_{E_k} f(x) \frac{\lambda_k(dx)}{(\sqrt{2\pi hi})^k}, \tag{2}$$

and extend this by putting convergent factors as

$$F_k(f) = \lim_{n\to\infty} \int_{E_k} f(x) \exp\left[-\frac{1}{2n} (V^{-1}(x - a), (x - a))\right] \frac{\lambda_k(dx)}{(\sqrt{2\pi hi})^k} \tag{3}$$

where a is any element of E_k and V is a strictly positive-definite symmetric operator. The domain $\mathfrak{D}(F_k)$ of definition of F_k is the space of all Borel measurable functions for which the limit in (3) exists for every (a, V) and has a finite value independent of (a, V). We shall rewrite (3) as

$$F_k(f) = \lim_{n\to\infty} \prod_{\nu=1}^{k} \sqrt{1 + \frac{nv_\nu}{hi}} \int_{E_k} f(x) N(dx\colon a, nV), \tag{3'}$$