RANDOM SHIFTS OF STATIONARY PROCESSES

HERMANN DINGES
MATHEMATISCHES INSTITUT DER TECHNISCHEN HOCHSCHULE, MÜNCHEN

1. Introduction

A variety of results concerning strongly stationary processes with smooth trajectories turns out to be derivable from a theorem, which extends the formula for changing the variable in the differential on the real axis to the case of measure spaces with a one-parameter group of measure preserving transformations.

The paper starts with the statement of three results, which will be shown in the end to be special cases of the main theorem. The middle part consists of the formulation and proof of this main theorem.

EXAMPLE I. Let x_t, $t \in R$ be an E-valued measurable stationary process, that is, a Lebesgue-measurable family of mappings from a measure space $(\Omega, \mathfrak{M}, P)$ into a measure space (E, B). The σ-algebra $\mathfrak{M}$ is generated by the x_t, and P is a σ-finite measure such that the shift transformations S_u, $u \in R$, leave P unchanged; that is,

$$(1) \qquad PS_u(x_{t_1} \in B_1, \cdots, x_{t_n} \in B_n) = P(x_{t_1+u} \in B_1, \cdots, x_{t_n+u} \in B_n) = P(x_{t_1} \in B_1, \cdots, x_{t_n} \in B_n),$$

and therefore,

$$(2) \qquad PS_u(A) = P(A), \qquad \text{for all} \quad A \in \mathfrak{M}.$$

Assume now that E is the real axis and that for almost all ∞, $\omega \in \Omega$, $x(t, \omega) = x_t(\omega)$ is a differentiable function of t with derivative $\dot{x}(t, \omega)$. Let $t = h(x)$ be a differentiable function such that for almost all ω the graph in $R \times R$ of $t \to x(t, \omega)$ has exactly one point in common with the graph of $x \to h(x)$. This common point will be called $(t(\omega), x(t(\omega), \omega))$; $h(x(t(\omega), \omega)) = t(\omega)$. The most trivial example is $h = t^* =$ const. If $\dot{x}(t, \omega)$ is bounded from above for all paths of a process, then every h with sufficiently small positive derivative would do.

Now let h be fixed. We write $(x(\omega), \dot{x}(\omega))$ which is short for $(x(t(\omega), \omega), \dot{x}(t(\omega), \omega))$. We consider the "shift by h" S_h defined as follows: $S_h(\omega)$ belongs to the set $\{x_u \in B\}$ if and only if $x(t(\omega) + u, \omega) \in B$. The result that interests us here is

$$(3) \qquad (P)S_h = P(1 - \dot{x}(0, \omega)h'(x(0, \omega)))$$

or

$$(4) \qquad (P(1 - \dot{x}(\omega)h'(x(\omega)))^{-1})S_h = P.$$