ON THE EXISTENCE OF A LIFTING COMMUTING WITH THE LEFT TRANSLATIONS OF AN ARBITRARY LOCALLY COMPACT GROUP

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1. Introduction

The main purpose of this paper is to establish the existence of a lifting commuting with the left translations of an arbitrary locally compact group. The material is divided into nine sections and two appendices. The second section contains the notations and terminology used throughout the paper. The third one contains several preliminary results and remarks. In sections 4 and 5 we define and study the conditional expectation P_H arising from a quotient group. In sections 6 and 7 we give various results concerning liftings, and in particular, we study the problem of extending a lifting "from a quotient group to the group." The main results of this paper are given in sections 8 and 9. Appendix I contains various remarks on adequate families of measures. In appendix II we prove a maximal ergodic theorem.

2. Notations and terminology

Let Z be a locally compact space. As usual, we denote by $C^\infty(Z)$ the algebra of all bounded real-valued continuous functions on Z and by $\mathfrak{K}(Z)$, the subalgebra of $C^\infty(Z)$ consisting of all $f \in C^\infty(Z)$ having compact support. We use the notation $\mathfrak{M}(Z)$ for the vector space of all real Radon measures on Z and the notation $\mathfrak{M}_+(Z)$ for the cone of all positive Radon measures on Z.

Now let $\mu \in \mathfrak{M}_+(Z)$, $\mu \neq 0$. As usual, we denote by $\mathfrak{L}(Z, \mu)$ the algebra of all real-valued μ-measurable functions on Z and by $\mathfrak{N}(Z, \mu)$, the ideal of all $f \in \mathfrak{L}(Z, \mu)$ which are locally μ-negligible. For $f, g \in \mathfrak{L}(Z, \mu)$ we write $f \equiv g(\mu)$, if f and g coincide locally almost everywhere with respect to μ, that is, if $f - g \in \mathfrak{N}(Z, \mu)$. We denote by $f \to \tilde{f}$ the canonical mapping of $\mathfrak{L}(Z, \mu)$ onto the quotient algebra $\mathfrak{L}(Z, \mu)/\mathfrak{N}(Z, \mu)$.

For a real-valued function g which is defined on the complement of a locally μ-negligible set and is μ-measurable, we agree to call equivalence class of g and

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